Markov Mills, Reliable Rolls and Monte-Carlo Mines: Minimizing the Operating Costs of Grinding Mills

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We develop, analyze and compare several models for the process of ordering and replacing rolls of grinding mills in a mine, including a Markov model, a reliability engineering based model and a Monte-Carlo simulation. The purpose of the models is to help reduce the cost of mill outages and the commissioning and storing of the mill rolls.

1 Introduction

In a mineral mine, ore is extracted from the earth, crushed and ground and then further processed and refined. It is with the crushing and grinding step of the process, called comminution (the mechanical process of rock breakage, excluding blasting), that this paper is concerned. The primary equipment for this activity are grinding mills, of which there are many types based on different design principles. In the mine we analyzed, such a mill consists of two large rolls which, by rotating in opposite directions, crush the ore between them. Since each mill in the mine processes a huge amount of material and because equipment of this size and grade is expensive (a single roll can cost on the order of magnitude of 10^5 €), both mill outages and the commissioning and storing of rolls cause enormous costs. It is therefore natural to ask how, when and how many rolls one should order to minimize the operating costs of the mine's grinding process.

The paper is structured as follows: After the introduction to the topic, we give a description of the problem; then, a Markov model, a reliability engineering based model and a Monte-Carlo Simulation are described, together with their results. Last, we give a comparison of the models and future prospects.

2 Problem Description

A mill *fails* whenever one or both of it's rolls fail, mostly due to attrition. It can be repaired by using two new rolls; this holds even if only one roll fails because the two rolls of a mill have to be in the same state of abrasion. If a roll fails due to attrition, it can sometimes be renewed once;[1] we do not model this. A mill that is not working causes *downtime costs* $c_d(t) = c_{\text{down}} * \#_{\text{defect}}(t)$, where c_{down} are the downtime costs for one mill per day and $\#_{\text{defect}}(t)$ is the number of defect mills on day t.

New rolls can be ordered anytime and arrive at the mill after the replacement time has passed. They can be stored in an inventory if necessary. If rolls stay in inventory, they cause *inventory costs* $c_i(t) = \#_{\text{inv}}(t) * c_{\text{roll}} * p$, where $\#_{\text{inv}}(t)$ is the number of rolls in inventory on day t and $p \in [0, 1]$ is the inventory cost factor.[2] At the time of ordering, a fixed percentage $q \in [0, 1]$ of the roll cost c_{roll} is paid in advance; the remainder $(1 - q)c_{\text{roll}}$ is paid upon delivery. Advance payments cause the *prefinance*

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[1] The roll can only be renewed once because the procedure involves a reduction of the roll diameter.

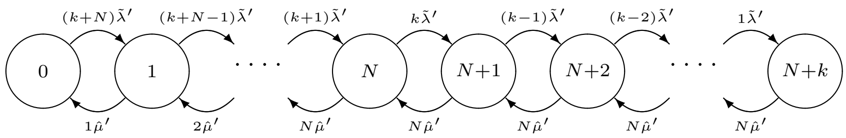
[2] The inventory cost factor expresses the total inventory costs per roll and day. It includes facility and personnel costs as well as interest and obsolescence of the roll.

costs $c_f(t) = \#_{\text{ordered}}(t) * c_{\text{roll}} * q * r$, where $\#_{\text{ordered}}(t)$ is the number of rolls commissioned on day t and $r \geq 0$ is the interest rate per day used to value the advance. The total cost $c(t) = c_d(t) + c_i(t) + c_f(t)$ is the sum of all the costs above.

We assume iid roll lifetimes and iid replacement times. By μ and σ ($\hat{\mu}$ and $\hat{\sigma}$) we denote expectation and standard deviation of the roll (mill) lifetime distribution,[3] by λ ($\tilde{\lambda}$) the expectation of the replacement time distribution for a roll (pair of rolls),[4] and by N the number of mills in the mine.

3 Markov Mills

A simple ordering strategy is to have a fixed-size inventory, where a new pair of rolls is ordered each time a pair of rolls is taken out. Broken mills are repaired as long as $\#_{\text{inv}} \geq 2$; if a mill breaks and the inventory is empty, a new pair of rolls is ordered, too. This simple strategy lends itself well to a Markov model. We keep track of $\#_{\text{inv}}/2$ and $N - \#_{\text{defect}}$ with the following states: For states $0, 1, \ldots, N$, the inventory is empty and the state index gives the number of working mills, whereas for states $N+1, N+2, \ldots, N+k$ all mills are intact and the state index minus N gives the number of roll pairs in inventory. For notational convention, we abbreviate rates with $\hat{\mu}' := 1/\hat{\mu}$ and $\tilde{\lambda}' := 1/\tilde{\lambda}$.



Since we are working with a memoryless model, we assume exponential distributions for both mill lifetimes and roll pair replacement times.[5] To calculate the cost function $c(t)$ in terms of $\hat{\mu}$ and $\tilde{\lambda}$ for $t \to \infty$, we obtain the equilibrium distribution π by solving

$$\begin{aligned}
\pi_0 \cdot (k+N)\tilde{\lambda}' &= \pi_1 \cdot \hat{\mu}' \\
\pi_{i-1} \cdot (k+N-i+1)\tilde{\lambda}' + \pi_{i+1} \cdot (i+1)\hat{\mu}' &= \pi_i \cdot (k+N-i)\tilde{\lambda}' + \pi_i \cdot i\hat{\mu}' && 0 < i < N \\
\pi_{N+i-1} \cdot (k-i+1)\tilde{\lambda}' + \pi_{N+i+1} \cdot N\hat{\mu}' &= \pi_{N+i} \cdot (k-i)\tilde{\lambda}' + \pi_{N+i} \cdot N\hat{\mu}' && 0 \leq i < k \\
\pi_{N+k-1} \cdot \tilde{\lambda}' &= \pi_{N+k} \cdot N\hat{\mu}'
\end{aligned}$$

under the constraint $\sum_{i=0}^{N+k} \pi_i = 1$. A short computation yields

$$\pi_i = \pi_0 \binom{k+N}{i} \left(\frac{\tilde{\lambda}'}{\hat{\mu}'}\right)^i, \quad 0 \leq i < N \qquad \pi_{N+i} = \pi_0 \binom{k+N}{N} \binom{N}{i} \left(\frac{\tilde{\lambda}'}{\hat{\mu}'}\right)^{N+i} N^{-i} i!, \quad 0 \leq i \leq k$$

with $\pi_0 = \left(\sum_{i=0}^{N-1} \binom{k+N}{i} \left(\frac{\tilde{\lambda}'}{\hat{\mu}'}\right)^i + \binom{k+N}{N} \left(\frac{\tilde{\lambda}'}{\hat{\mu}'}\right)^N \sum_{i=0}^{k} \binom{N}{i} \left(\frac{\tilde{\lambda}'}{\hat{\mu}'}\right)^i N^{-i} i! \right)^{-1}$. With $\#_{\text{defect}} = \max(N-i, 0)$, $\#_{\text{inv}} = 2\max(i-N, 0)$ and $\#_{\text{ordered}} = 2(N+k-i)$, where i is the state index, these equations allow the computation of the total cost c for given $k, N, \lambda, \mu, c_{\text{roll}}, c_{\text{down}}, p, q, r$ in the Markov model.

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[3] The mill lifetimes can be computed from the roll lifetimes as follows: Let X, Y be iid random variables with density f and distribution F. Then $\hat{F}(x) := \mathbb{P}[\min(X,Y) \leq x] = \mathbb{P}[X \leq x \vee Y \leq x] = 1 - \mathbb{P}[X > x \wedge Y > x] = 1 - (1-F(x))^2 = 2F(x) - F^2(x)$, $\hat{f}(x) = \hat{F}'(x) = 2f(x)(1-F(x))$ and $\hat{\mu} = \int x\hat{f}(x)dx = 2\left(\mu - \int xf(x)F(x)dx\right)$.

[4] The value $\tilde{\lambda}$ can be computed in a way similar to $\hat{\mu}$, but with a maximum instead of a minimum: Let X, Y be iid random variables with density f and distribution F. Then $\tilde{F}(x) := \mathbb{P}[\max(X,Y) \leq x] = \mathbb{P}[X \leq x \wedge Y \leq x] = F^2(x)$, $\tilde{f}(x) = 2f(x)F(x)$ and $\tilde{\lambda} = 2\int xf(x)F(x)dx$.

[5] For exponential distributions, $\hat{\mu} = \frac{1}{2\mu}$, $\hat{\sigma}^2 = (\frac{1}{2\mu})^2$ and $\tilde{\lambda} = 3\frac{1}{2\lambda}$ with variance $5(\frac{1}{2\lambda})^2$.

4 Roll Reliability

Another ordering strategy is to time the commissioning of the rolls so that they arrive when they are needed, i. e. at the time of the next mill failure. Since roll failures are random, the rolls arrive in time (before the mill failure) only with a certain probability. This probability can be increased by ordering earlier, decreasing the chance for downtimes but increasing ordering related and inventorial costs. We model this with a *caution factor* α and order a pair of rolls at time $\hat{\mu} - \alpha\hat{\sigma} - \tilde{\lambda}$ after a mill failure, so that in expectation the rolls arrive $\alpha\hat{\sigma}$ days before the mill fails. This works well for $\hat{\mu} \geq \tilde{\lambda} + \alpha\hat{\sigma}$. In practice, however, the replacement time alone can exceed the lifetime of a mill. Therefore, we order at time $2\hat{\mu} - \sqrt{2}\alpha\hat{\sigma} - \tilde{\lambda}$ for $\hat{\mu} - \alpha\hat{\sigma} < \tilde{\lambda} \leq 2\hat{\mu} - \sqrt{2}\alpha\hat{\sigma}$. We call this a *lookahead* of two; in general, for a lookahead of i we order at time $i\hat{\mu} - \sqrt{i}\alpha\hat{\sigma} - \tilde{\lambda}$.[6]

For the following reliability considerations we restrict ourselves to a lookahead of one, normally distributed mill lifetimes with mean $\hat{\mu}$ and variance $\hat{\sigma}^2$ as well as a deterministic roll pair replacement time $\tilde{\lambda}$. Additionally, a *reserve* of size $k \in \mathbb{N}_0$ is maintained: Whenever $\#_{\text{inv}} < 2k$, a new pair of rolls is ordered immediately (in addition to the pair ordered for the next mill failure). Furthermore, we only allow a mill to use the reserve and the roll pair ordered for it, i. e. we restrict the interactions between the mills in the failure/replacement process to the reserve. For given k, let m be the *mean time to repair*, i. e. the average time until a mill starts working again after failing, and set $w := \hat{\mu} - \alpha\hat{\sigma} - \tilde{\lambda}$. Let $\hat{F}$ be the lifetime distribution of a mill, $\tilde{G}$ the replacement time distribution for a pair of rolls, Φ the standard normal distribution and s the steady state probability that there is no roll pair in inventory. To compute the mill availability $\frac{\hat{\mu}}{\hat{\mu}+m}$, we express m via Lebesgue-Stieltjes integration [2] as

$$\begin{aligned} m &= s\int_0^\infty d\hat{F}(u)\int_{u-w}^\infty (w+v-u)\,d\tilde{G}(v) = s\int_0^{\hat{\mu}-\alpha\hat{\sigma}} (\hat{\mu}-\alpha\hat{\sigma}-u)\,d\hat{F}(u) \\ &= s(\hat{\mu}-\alpha\hat{\sigma})\hat{F}(\hat{\mu}-\alpha\hat{\sigma}) - s\int_0^{\hat{\mu}-\alpha\hat{\sigma}} u\,d\hat{F}(u) \leq s(\hat{\mu}-\alpha\hat{\sigma})\Phi(-\alpha) \end{aligned} \tag{1}$$

In the above equations we have made use of the facts that $\tilde{G}$ is deterministic with mean $\tilde{\lambda}$, $\hat{F}$ is normally distributed and

$$\int_{u-w}^\infty (w+v-u)\,d\hat{G}(v) = \begin{cases} 0 & \text{for } u-w \geq \tilde{\lambda} \iff u \geq \hat{\mu}-\alpha\hat{\sigma}, \\ \hat{\mu}-\alpha\hat{\sigma}-u & \text{for } u-w \leq \tilde{\lambda} \iff u \leq \hat{\mu}-\alpha\hat{\sigma} \end{cases}.$$

The term $-s\int_0^{\hat{\mu}-\alpha\hat{\sigma}} ud\hat{F}(u)$ in equation (1) can be expressed via the error function [4] to improve accuracy. To estimate the steady state probability s, we model the request for reserve roll pairs as a Poisson process [3] with demand rate $b := \frac{N}{\hat{\mu}}\Phi(-\alpha)$. The probability of the event that there will be k or more demands within the replacement time is then $s = 1 - e^{-b\tilde{\lambda}}\sum_{i=0}^{k-1}\frac{(b\tilde{\lambda})^i}{i!}$ and we obtain the following inequality for the mill availability:

$$\frac{\hat{\mu}}{\hat{\mu}+m} \geq \frac{\hat{\mu}}{\hat{\mu} + \left(1 - e^{-b\tilde{\lambda}}\sum_{i=0}^{k-1}\frac{(b\tilde{\lambda})^i}{i!}\right)(\hat{\mu}-\alpha\hat{\sigma})\Phi(-\alpha)}.$$

In expectation, there are $k - b\tilde{\lambda}$ demands for roll pairs from the reserve; of these, a fraction of $1-s$ can be satisfied. Since the number of roll pairs leaving and entering the reserve must be equal, in expectation there are $k - b\tilde{\lambda}(1-s)$ roll pairs in inventory due to the reserve;[7] together with the $N\alpha\hat{\sigma}/\hat{\mu}$ roll pairs due to arrivals before mill failures, we get $\#_{\text{inv}} = 2(k - \frac{N}{\hat{\mu}}(\Phi(-\alpha)\tilde{\lambda}(1-s) - \alpha\hat{\sigma}))$. In expectation, $N/\hat{\mu}$ mill failures happen per time unit and $\#_{\text{ordered}}/\tilde{\lambda}$ rolls arrive, so $\#_{\text{ordered}} = 2N\tilde{\lambda}/\hat{\mu}$. We can now compute an upper bound for the total cost c.

[6] The factor $\sqrt{i}$ accounts for the fact that for iid random variables X_i, $\sum_i \mathbb{V}(X_i) = \mathbb{V}(\sum_i X_i)$. [1]

[7] This term is always positive since $b\tilde{\lambda}\sum_{i=0}^{k-1} e^{-b\tilde{\lambda}}\frac{(b\tilde{\lambda})^i}{i!} = \sum_{i=1}^{k} e^{-b\tilde{\lambda}}\frac{(b\tilde{\lambda})^i}{i!} i \leq k\sum_{i=1}^{k} e^{-b\tilde{\lambda}}\frac{(b\tilde{\lambda})^i}{i!} < k$.

5 Monte-Carlo Mine

The advantage of a simulation when compared with analytical models is that one has to rely less on simplifying assumptions. The simulation is also easier to adapt to problem modifications. On the other hand, it's more difficult to prove properties and correctness. We have implemented a Monte-Carlo simulation which allows for arbitrary lookahead, roll lifetime and replacement time distributions.

The lookahead is determined by finding $\min\{i \in \mathbb{N} | i\hat{\mu} - \sqrt{i}\alpha\hat{\sigma} - \tilde{\lambda} \geq 0\}$. The ordering strategy is the same as in the previous chapter, except that any mill may use any pair of rolls, no matter for which mill it was ordered. We do not present the details of the implementation here due to space limitations.

6 Comparison & Future Prospects

The three models introduced here[8] differ in their trade-off between the accuracy with which they model the real situation and their suitability for analysis. For the Markov model, the limiting factors are the assumption that lifetimes and replacement times are exponentially distributed and the ineffective ordering policy. The reliability based model allows arbitrary lifetime distributions and uses a more efficient ordering policy; it, too, allows analytic statements, but is limited to a lookahead of one and deterministic replacement times. The Monte-Carlo simulation offers the highest degree of flexibility by allowing arbitrary distributions and ordering policies, at the price of aggravating analysis. For practical purposes, we use the Monte-Carlo simulation for results and the analytic models to cross-validate these results. The table below gives the results of the reliability model and the simulation for one dataset.

Number	Reliability	Simulation
$N - \#_{\text{defect}}$	1.99	1.99
$\#_{\text{inv}}$	0.53	0.44
$\#_{\text{ordered}}$	3.26	3.24

Table 1 Results of the reliability based model and the simulation for the input parameters $N = 2$, $k = 0$, $\alpha = 1.5$, $\hat{\mu} = 475$, $\hat{\sigma} = 47.5$ and $\tilde{\lambda} = 365$. The accuracy of the mill availability figure for the reliability-based model has been improved by expressing the remaining integral in equation (1) via the error function.

There are several directions in which our approach can be extended: The models can be expanded to better fit the real situation, at the cost of complicating analysis; this could include other ordering strategies, different distributions, e. g. lognormal or Weibull distribution, a more complex cost model, the renewal of rolls, redundancy considerations etc. Another direction is to consider other mill components and their interactions with each other, too. While it is probably not worthwhile to take cheap parts into account, the number of important (i. e. expensive) parts in a grinding mill is on the order of magnitude 10^1.

Finally, we hope that this paper has given a convincing and maybe even entertaining example of the applicability of stochastics in industry.

References

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[8] We also developed a model based on queuing theory, which we do not present here due to space limitations.